EE 435

Lecture 27

Data Converter Characterization

- Linearity Metrics
- Spectral Characterization

Review From Last Lecture

INL-based ENOB

Consider initially the continuous INL definition for an ADC where the INL of an ideal ADC is $X_{LSB}/2$

Assume
$$INL = \theta X_{REF} = \upsilon X_{LSBR}$$

where X_{LSBR} is the LSB based upon the defined resolution

Define the effective LSB by
$$\mathcal{X}_{LSBEFF} = \frac{\mathcal{X}_{REF}}{2^{n_{EQ}}}$$

Thus
$$INL = \theta 2^{n_{EQ}} X_{LSBEFF}$$

Since an ideal ADC has an INL of $X_{LSB}/2$, express INL in terms of ideal ADC
$$INL = \left[\theta 2^{(n_{EQ}+1)}\right]\left(\frac{X_{LSBEFF}}{2}\right)$$

Setting term in [ ] to 1, can solve for n_{EQ} to obtain
$$ENOB = n_{EQ} = \log_2\left(\frac{1}{2\theta}\right) = n_R - 1 - \log_2(\upsilon)$$

where n_R is the defined resolution

Review From Last Lecture

INL-based ENOB

$$ENOB = n_R - 1 - \log_2(\upsilon)$$

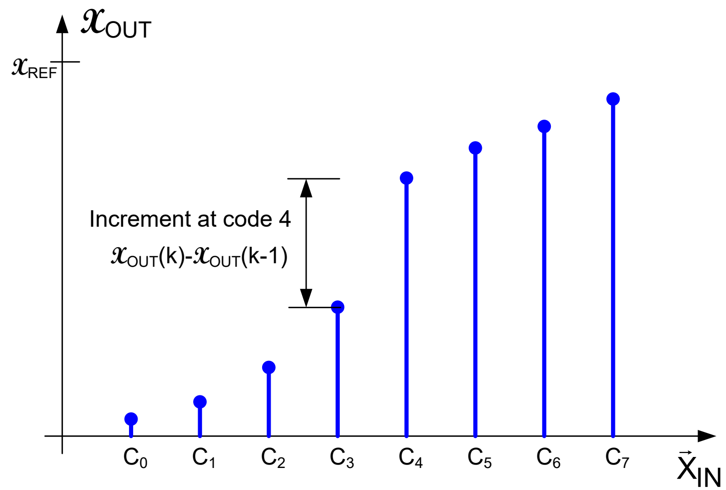
Consider an ADC with specified resolution of n_R and INL of v LSB

υ	ENOB
½	n
1	n-1
2	n-2
4	n-3
8	n-4
16	n-5

Review From Last Lecture

Differential Nonlinearity (DAC)

Nonideal DAC



Increment at code k is a signed quantity and will be negative if $X_{OUT}(k) < X_{OUT}(k-1)$

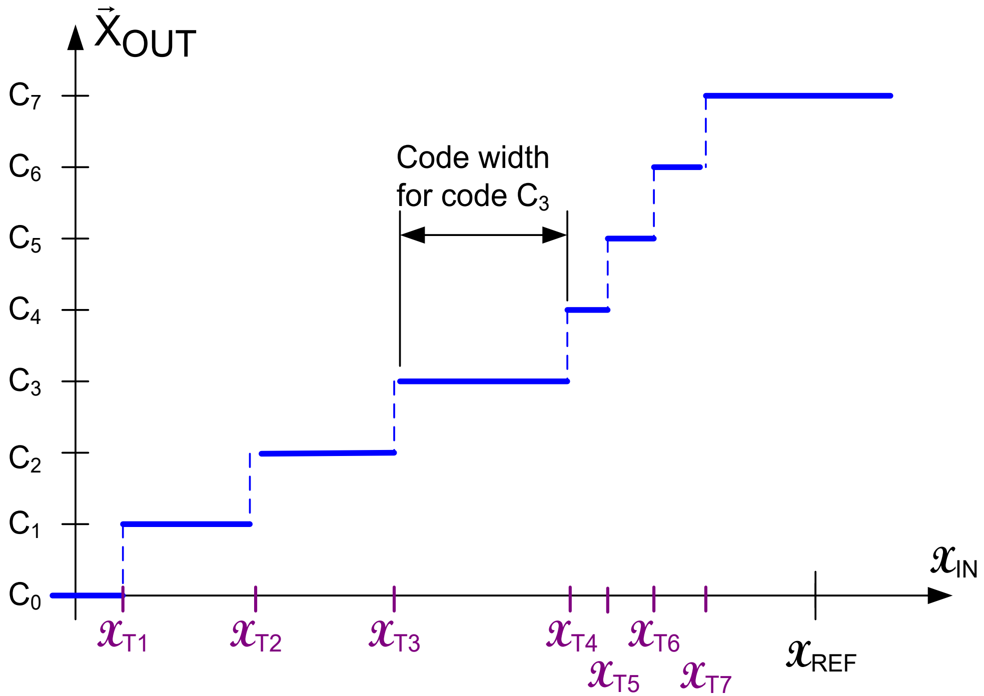
$$DNL(k) = \frac{X_{OUT}(k) - X_{OUT}(k-1) - X_{LSB}}{X_{LSB}}$$

$$DNL = \max_{1 \le k \le N-1} \{|DNL(k)|\}$$

DNL=0 for an ideal DAC

Differential Nonlinearity (ADC)

Nonideal ADC

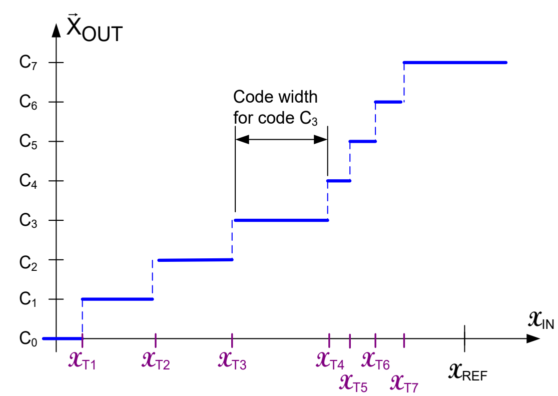


DNL(k) is the code width for code k – ideal code width normalized to X_{LSB}

$$DNL(k) = \frac{x_{T(k+1)} - x_{Tk} - x_{LSB}}{x_{LSB}}$$

Differential Nonlinearity (ADC)

Nonideal ADC



$$\mathrm{DNL}(k)=\frac{x_{T(k+1)}-x_{Tk}-x_{LSB}}{x_{LSB}}$$

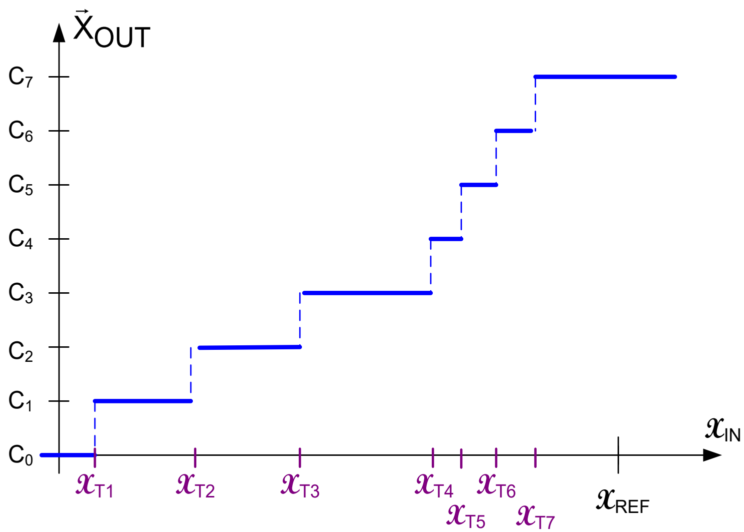
$$\mathrm{DNL}=\max_{2\le k\le N-1}\left\{\left|\mathrm{DNL}(k)\right|\right\}$$

DNL=0 for an ideal ADC

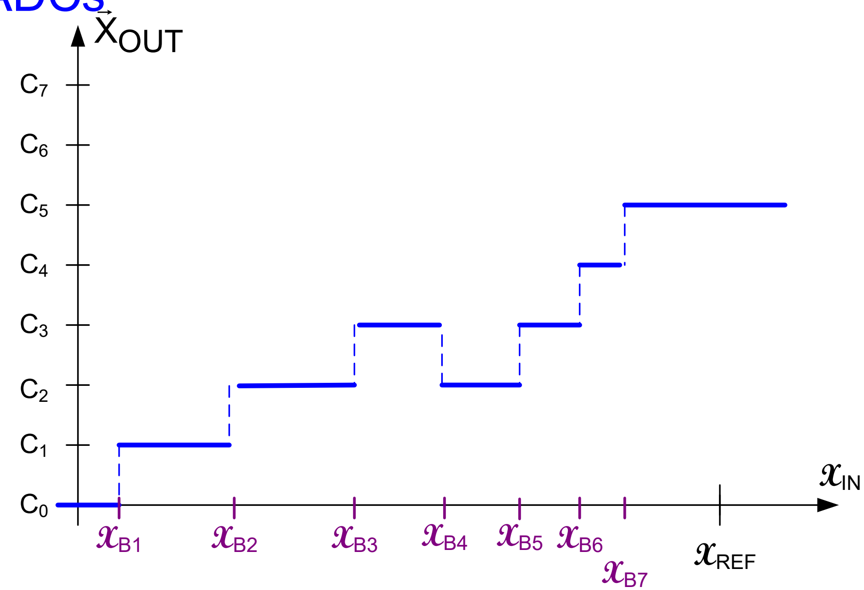
Note: In some nonideal ADCs, two or more break points could cause transitions to the same code C_k making the definition of DNL ambiguous

Monotonicity in an ADC

Nonideal ADCs



Monotone ADC



Nonmonotone ADC

Definition: An ADC is monotone if the

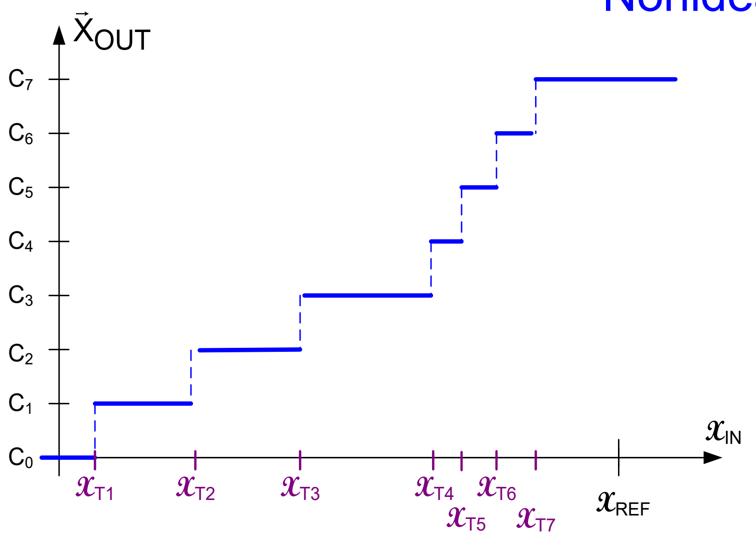
$$\vec{X}_{OUT}(x_k) \geq \vec{X}_{OUT}(x_m) \quad \text{whenever} \quad x_k \geq x_m$$

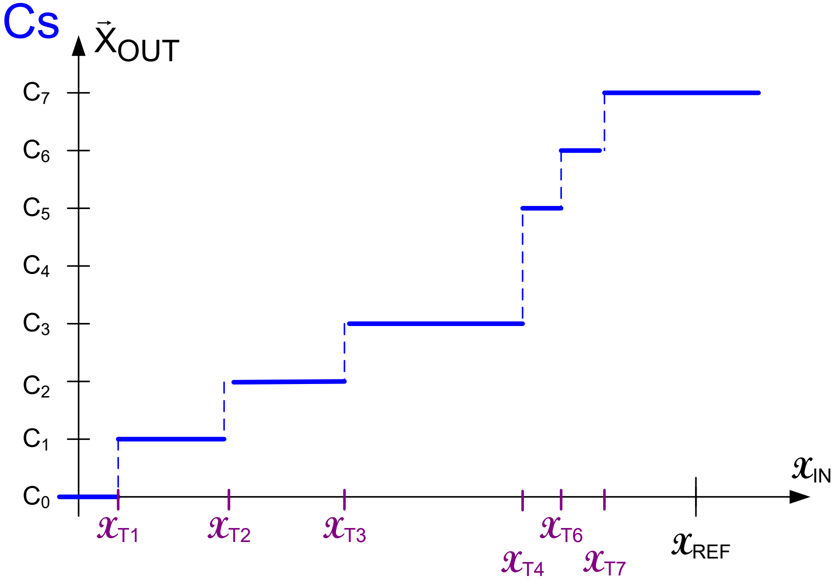
Note: Have used x_{Bk} instead of x_{Tk} since more than one transition point to a given code

Note: Some authors do not define monotonicity in an ADC.

Missing Codes (ADC)

Nonideal ADCs





No missing codes | One missing code

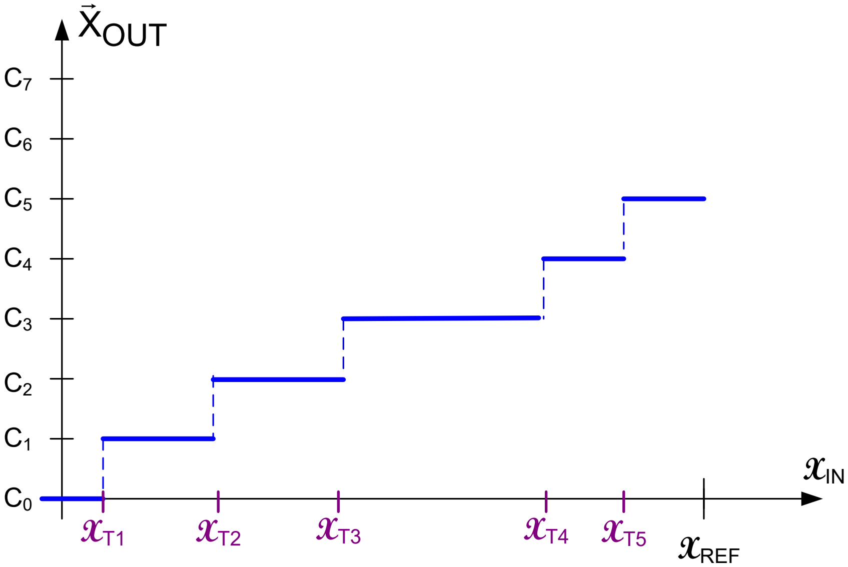
Definition: An ADC has no missing codes if there are N-1 transition points and a single LSB code increment occurs at each transition point. If these criteria are not satisfied, we say the ADC has missing code(s).

Note: With this definition, all codes can be present but we still say it has "missing codes"

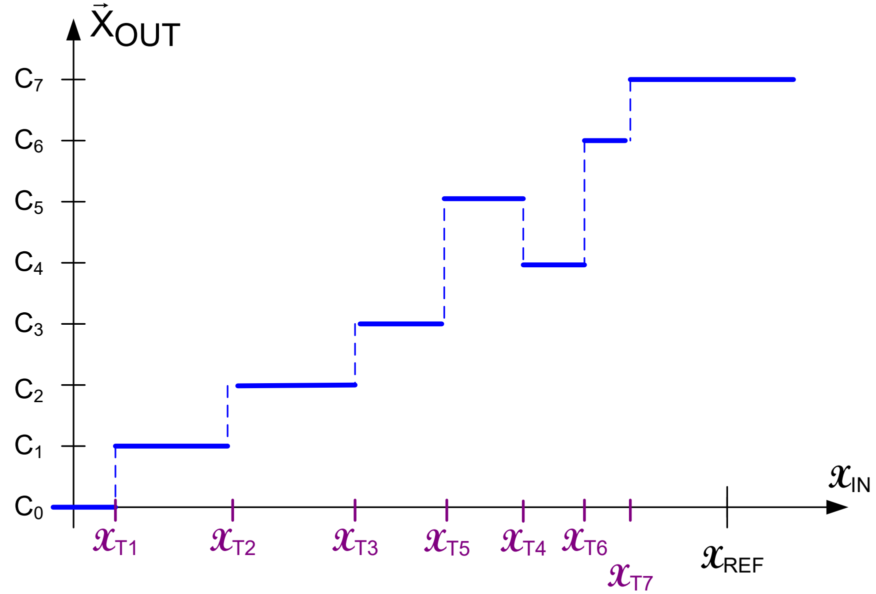
Note: Some authors claim that missing codes in an ADC are the counterpart to nonmonotonicity in a DAC. This association is questionable.

Missing Codes (ADC)

Nonideal ADCs



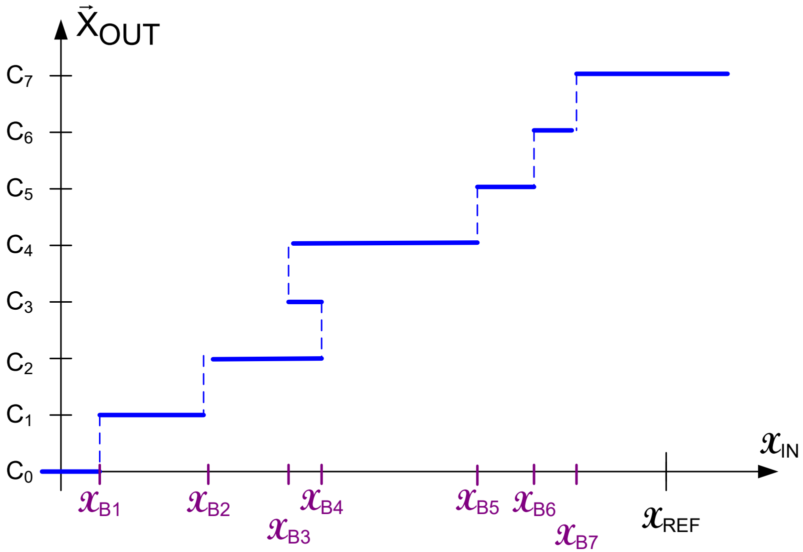
Missing codes

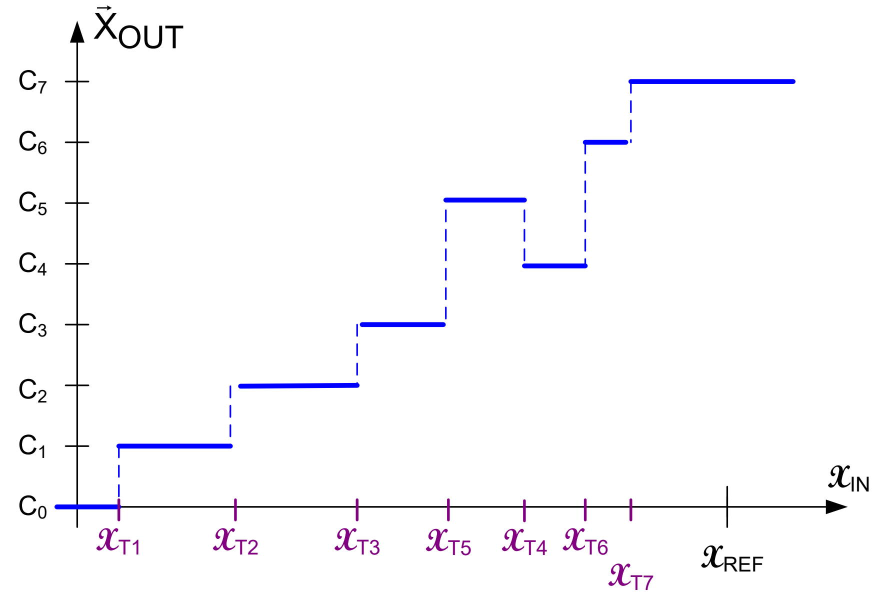


Missing code with all codes present

Weird Things Can Happen

Nonideal ADCs





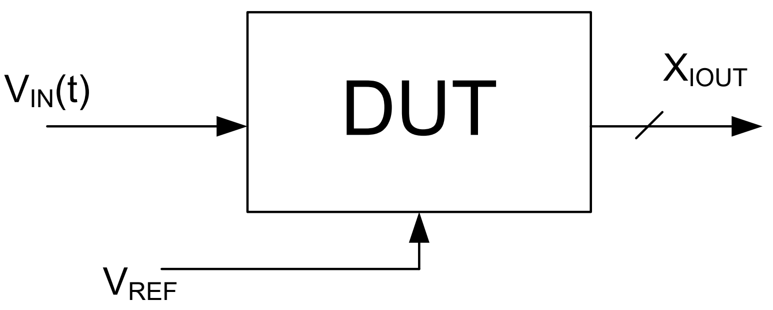
- Multiple outputs for given inputs
- All codes present but missing codes

Be careful on definition and measurement of linearity parameters to avoid having weird behavior convolute analysis, simulation or measurements

Most authors (including manufacturers) are sloppy with their definitions of data converter performance parameters and are not robust to some weird operation

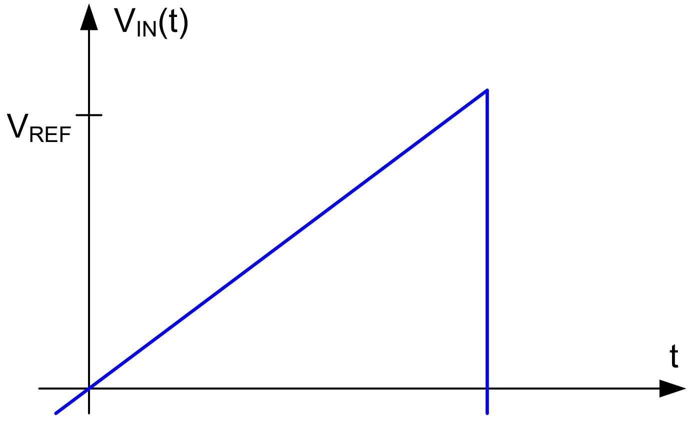
Linearity Measurements (testing)

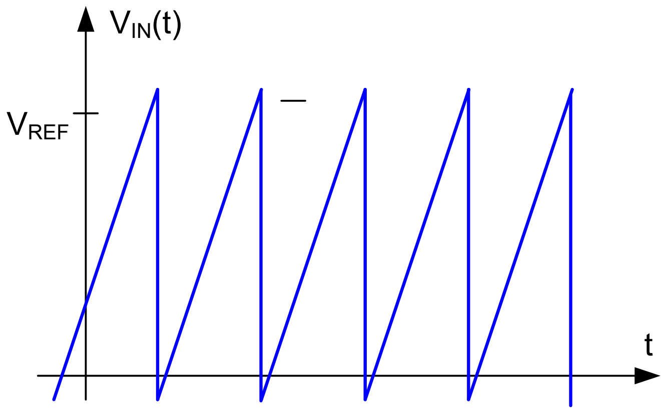
Consider ADC



Linearity testing often based upon code density testing

Code density testing:

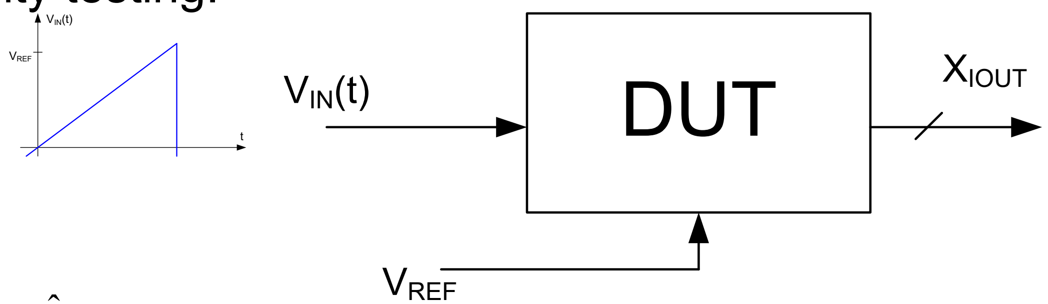


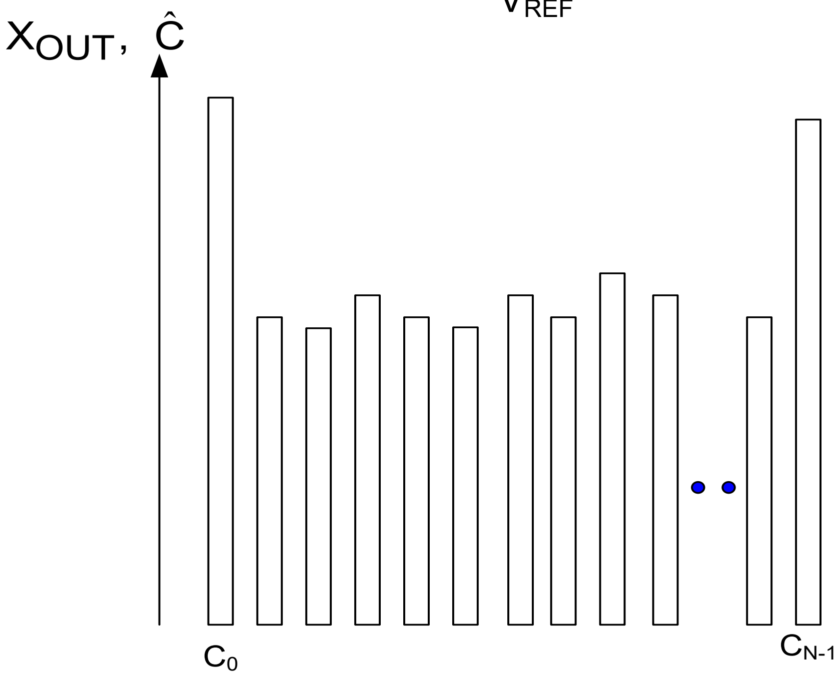


Ramp or multiple ramps often used for excitation
Linearity of test signal is critical (typically 3 or 4 bits more linear than DUT)

Linearity Measurements (testing)

Code density testing:



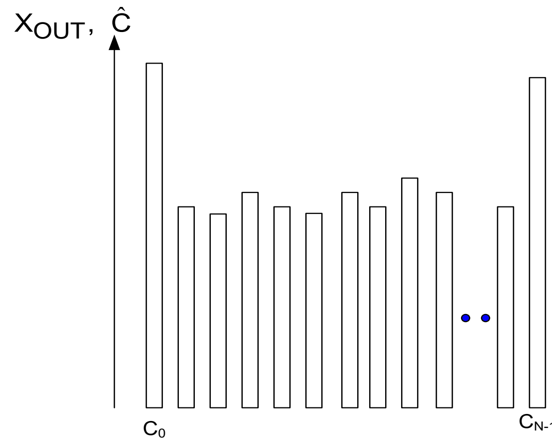


$\hat{C}_i$:number of counts in bin "i"

- First and last bins generally have many extra counts (and thus no useful information)
- Typically average 16 or 32 hits per code

Linearity Measurements (testing)

Code density testing:



$$\bar{C}=\frac{\sum_{i=1}^{N-2}\hat{C}_i}{N-2}$$

$$DNL_i=\frac{\hat{C}_i-\bar{C}}{\bar{C}}$$

$$INL_i=\begin{cases}0 & i=0,N-2\\ \dfrac{\left[\sum_{k=1}^{i}\hat{C}_k\right]-i\bar{C}}{\bar{C}} & 1\le i\le N\text{-}3\end{cases}$$

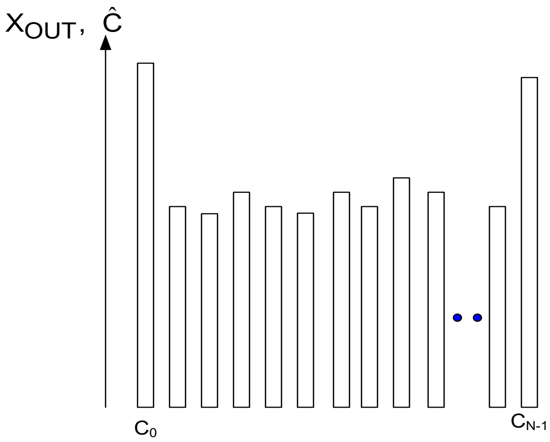
$$DNL=\max_{1\le i\le N-2}\{|DNL_i|\}$$

$$INL=\max_{1\le i\le N-3}\{|INL_i|\}$$

- This measurement is widely used
- Does not keep track of order bins are filled
- Some weird things can occasionally happen with this approach

Linearity Measurements (testing)

Code density testing:



$$\text{DNL} = \max_{1 \le i \le N-2} \left\{ \left| DNL_i \right| \right\}$$

$$\text{INL} = \max_{1 \le i \le N-3} \left\{ \left| INL_i \right| \right\}$$

Though INL and DNL for an ADC are rigorously defined, measuring the actual transition points is not practical even if n is small so code density tests are almost always used to “test” the INL and the DNL

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Linearity

A data converter (ADC or DAC) can be viewed as an amplifier that interfaces between the analog and digital domains

Linearity is of considerable concern in amplifiers irrespective of whether the I/O is analog:analog, analog:digital, digital:analog, or digital:digital

Though INL and DNL give some information about linearity (the term “linearity” is even included in their names!), much information about the actual linearity of a data converter is suppressed in the INL and DNL metrics

The seemingly simple concept of linearity is challenging to accurately characterize

Performance Characterization of Data Converters

- Static characteristics
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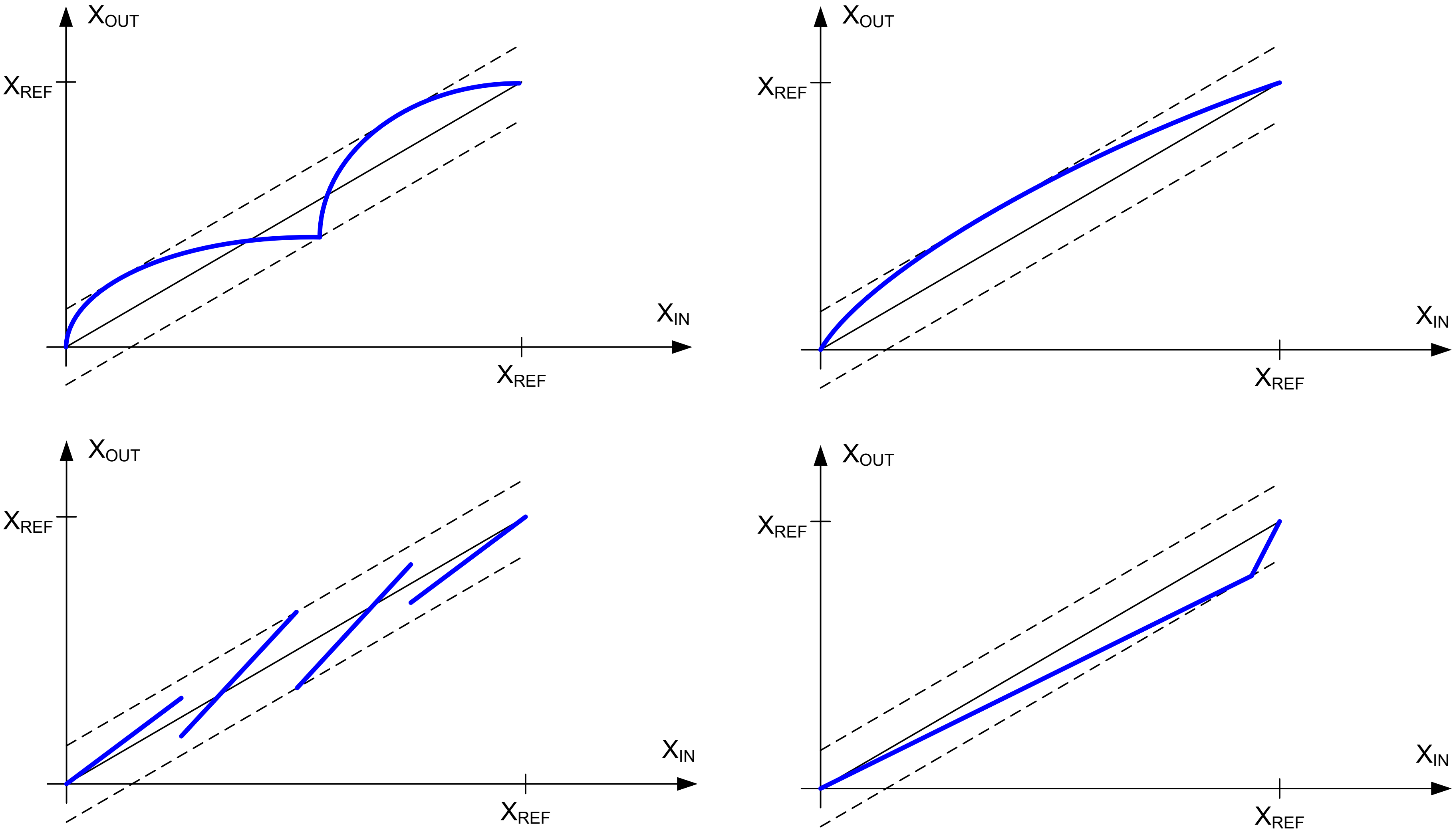
Linearity Metrics

Spectral Characterization

Spectral Characterization

INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity

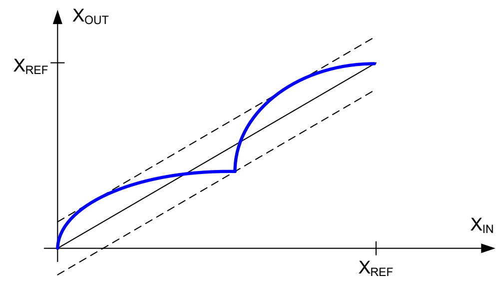


Linearity Issues

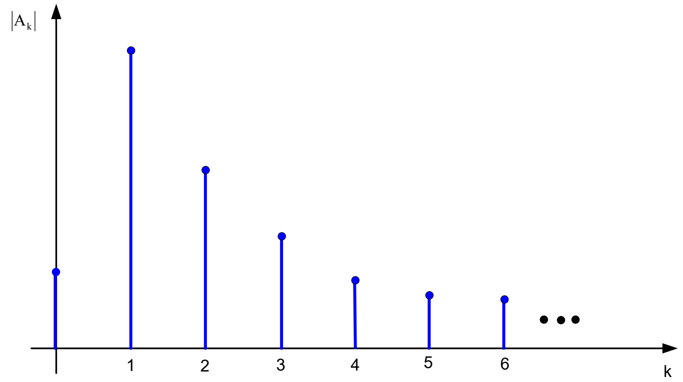
- INL is often not adequate for predicting the linearity performance of a data converter
- Distortion (or lack thereof) is of major concern in many applications
- Distortion is generally characterized in terms of the harmonics that may appear in a waveform when a periodic excitation is applied at the input

Two Popular Methods of Linearity Characterization

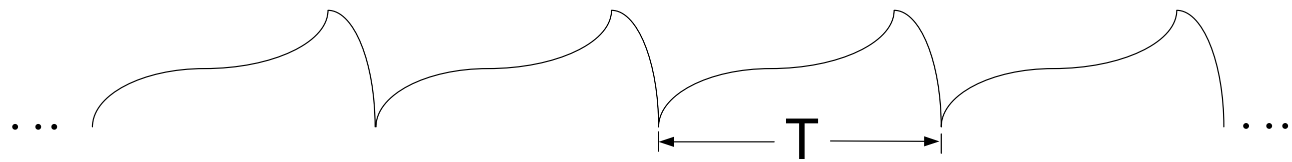
- Integral and Differential *Nonlinearity* (*metrics: INL,* DNL)



- Spectral Characterization (Based upon spectral harmonics of sinusoidal signals   metrics: THD, SFDR, SDR SNR)



Spectral Analysis



If f(t) is periodic

$$f(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin\left(k\omega t + \theta_k\right)$$

alternately

$$f(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin\left(k\omega t\right) + \sum_{k=1}^{\infty} b_k \cos\left(k\omega t\right) \qquad \omega = \frac{2\pi}{T}$$

$$A_k = \sqrt{a_k^2 + b_k^2}$$

Termed the Fourier Series Representation of f(t)

Metrics based upon Fourier Series Coefficients Useful for Characterizing how nonlinear a system is !

Fourier Series Representation of Periodic Continuous-Time Signals

$$f(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin\left(k\omega t + \theta_k\right)$$

Fourier Series Coefficients Determined From:

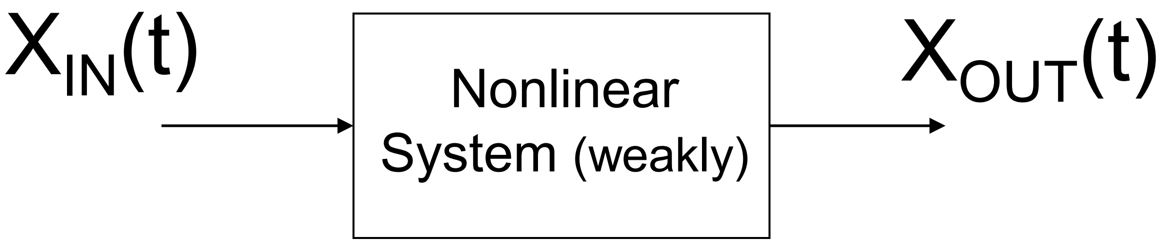
$$A_k = \frac{1}{\omega T}\left(\int_{t_1}^{t_1+T} f(t)e^{-jk\omega t}dt + \int_{t_1}^{t_1+T} f(t)e^{jk\omega t}dt\right)$$

or

$$a_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\sin(kt\omega)dt \qquad b_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\cos(kt\omega)dt$$

Integral is very time consuming, particularly if large number of components are required

Spectral Analysis



Often the system of interest is ideally linear but practically it is weakly nonlinear.

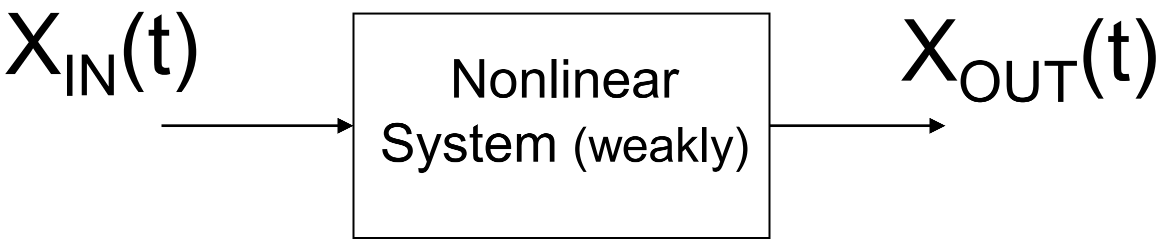
Often the input is nearly periodic and often sinusoidal and in latter case desired output is also sinusoidal

Weak nonlinearity will cause harmonic distortion (often just termed distortion) of signal as it is propagated through the system

Spectral analysis often used to characterize effects of the weak nonlinearity

Spectral Performance Dependent upon Magnitude and Offset of Input

Spectral Analysis



Distortion Types:

Frequency Distortion

Nonlinear Distortion (alt. harmonic distortion)

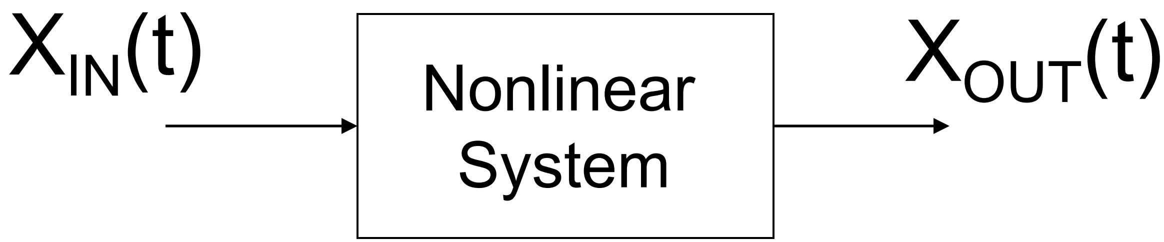
Frequency Distortion: Amplitude and phase of system is altered but output is linearly related to input (i.e. system remains linear)

Nonlinear Distortion: Characteristic of System that is not linear, frequency components usually appear in the output that are not present in the input

“Distortion” refers to two entirely different phenomenon

Spectral Analysis is the characterization of a system with a periodic input that relates the Fourier series relationships between the input and output waveforms

Spectral Analysis



If $$X_{IN}(t) = X_m \sin(\omega t + \theta)$$

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

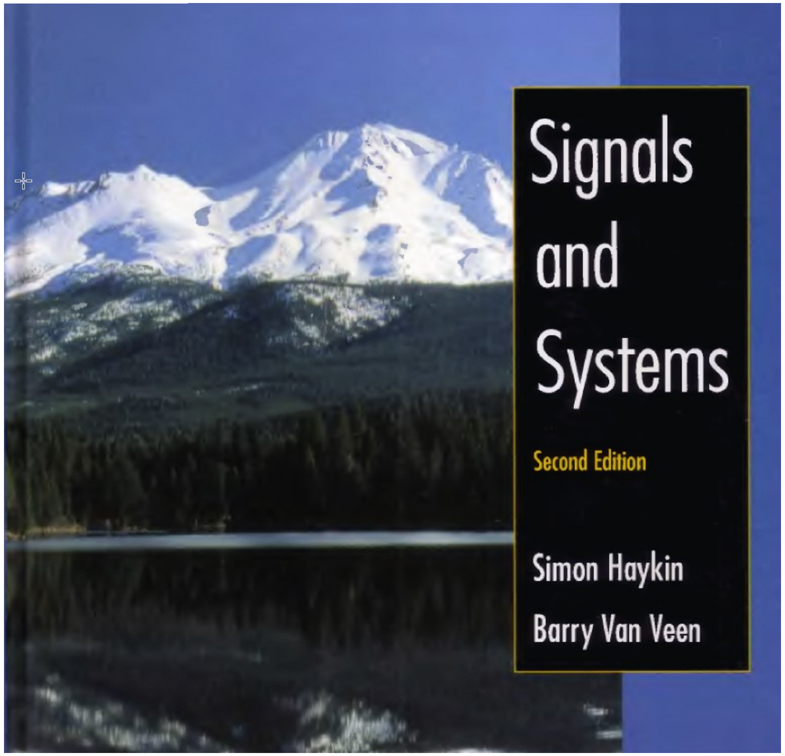
All spectral performance metrics depend upon the sequences $\langle A_k \rangle_{k=0}^{\infty}$ $\langle \theta_k \rangle_{k=1}^{\infty}$

Spectral performance metrics of interest: SNDR, SDR, THD, SFDR, IMOD

Alternately

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega t) + \sum_{k=1}^{\infty} b_k \cos(k\omega t) \qquad A_k = \sqrt{a_k^2 + b_k^2} \qquad \theta_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

3.3 *Fourier Representations for Four Classes of Signals*



There are four distinct Fourier representations, each applicable to a different class of signals. The four classes are defined by the periodicity properties of a signal and whether the signal is continuous or discrete in time. The Fourier series (FS) applies to continuous-time periodic signals, and the discrete-time Fourier series (DTFS) applies to discrete-time periodic signals. Nonperiodic signals have Fourier transform representations. The Fourier transform (FT) applies to a signal that is continuous in time and nonperiodic. The discrete-time Fourier transform (DTFT) applies to a signal that is discrete in time and nonperiodic. Table 3.1 illustrates the relationship between the temporal properties of a signal and the appropriate Fourier representation.

FS, FT, DTFS,DTFT

DFT (**Discrete Fourier Transform**) is a practical version of the **DTFT**, that is computed for a finite-length discrete signal. The **DFT** becomes equal to the **DTFT** as the length of the sample becomes infinite and the **DTFT** converges to the continuous Fourier transform **in the** limit of the sampling frequency going to infinity. Oct 27, 2014

The DFT is the most important discrete transform, used to perform Fourier analysis in many practical applications.[1] In digital signal processing, the

DFS, DTFT, and DFT

Herein we describe the relationship between the Discrete Fourier Series (DFS), Discrete Time Fourier Transform (DTFT), and the Discrete Fourier Transform (DFT). Why? The real reason is that the DFT is easily implemented on a computer and is part of every mathematics package, so it would be nice to know how to determine or approximate the DFT and DTFT on a computer.

Fast Fourier transform - Wikipedia

A **fast Fourier transform** (**FFT**) is an algorithm that computes the discrete Fourier transform (DFT) of a sequence, or its inverse (IDFT). Fourier analysis converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa.

DFT,DFS,FFT,IDFT

The “Fourier” Representations:

FS, FT, DTFS,DTFT

DFT, DFS, FFT, IDFT

Really fundamental concepts but varying notation and maybe varying perceptions

Spectral Characterization

Assume f(t) is periodic with period T and band-limited

f(t) is sampled N times at with sampling interval T_S $NT_s = T$

time domain $$f(t) = \sum_{i=1}^{N} A_k \sin(k\omega t + \theta_k)$$ 2N parameters

$$\vec{x} = < f(T_S), f(2T_S), \ldots . f(NT_S) >$$

(A_k, θ_k)

IDFT

DFT

frequency domain $$\vec{X} = < X_1, X_2,,,, X_N >$$ 2N parameters

(X_k are complex)

f(t)=IDFT (DFT(f(t)))

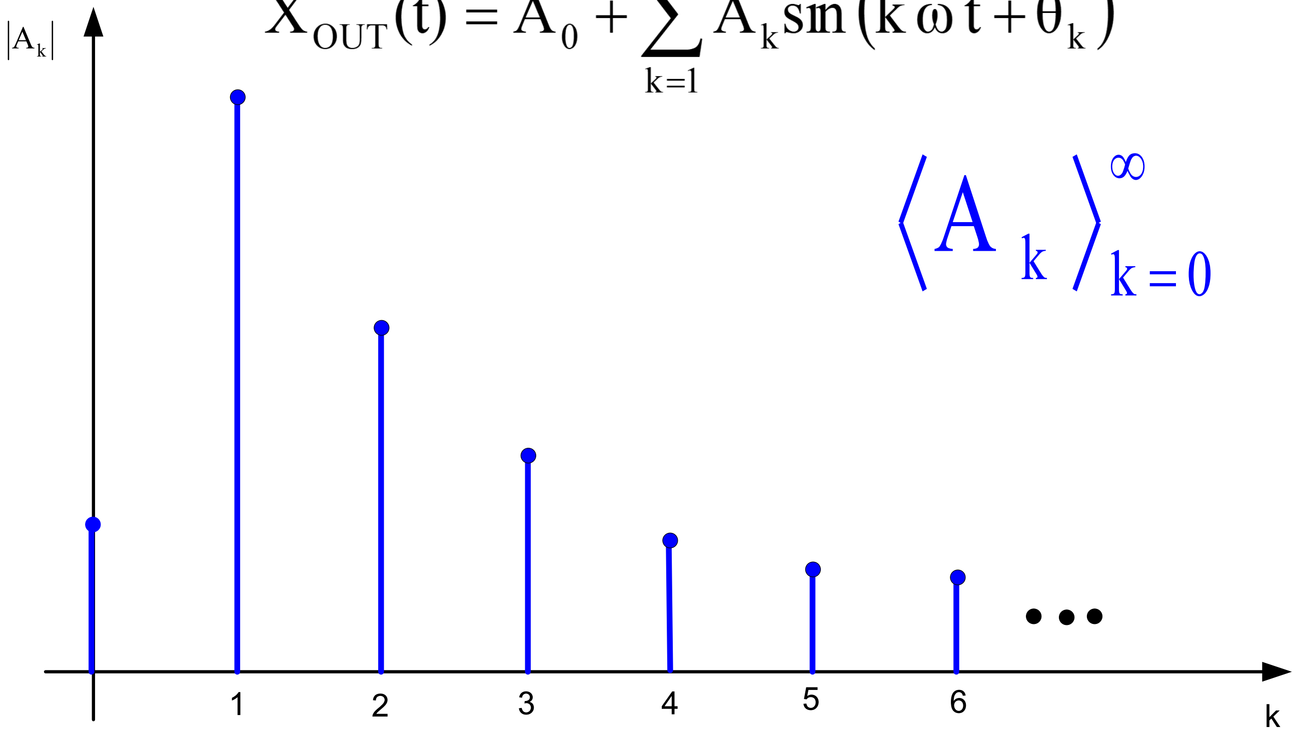
Spectral Characterization

Will focus on how Fourier Series Representation of a periodic signal is altered when it passes through a weakly nonlinear system

Relationship between DFT and continuous-time Fourier Series representation is fundamental to characterizing spectral performance of a weakly nonlinear system

Distortion Analysis

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

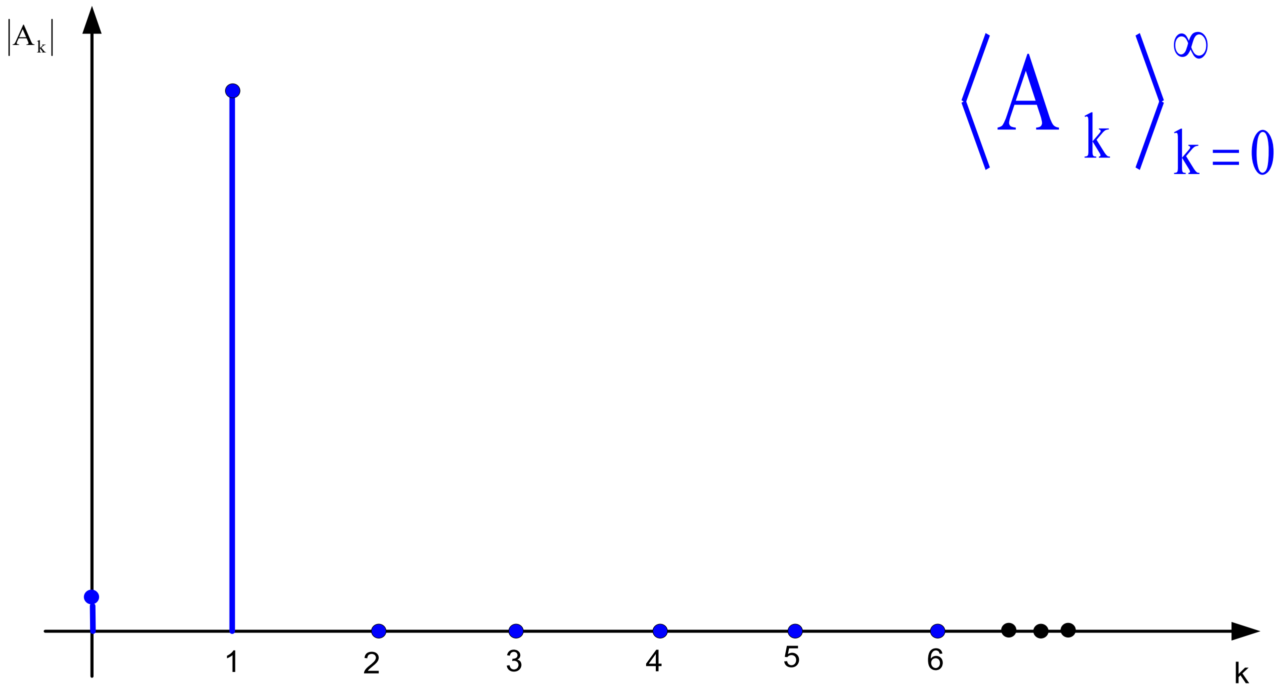


$$\langle A_k \rangle_{k=0}^{\infty}$$

- Often termed the DFT coefficients (will show later)
- Spectral lines, not a continuous function

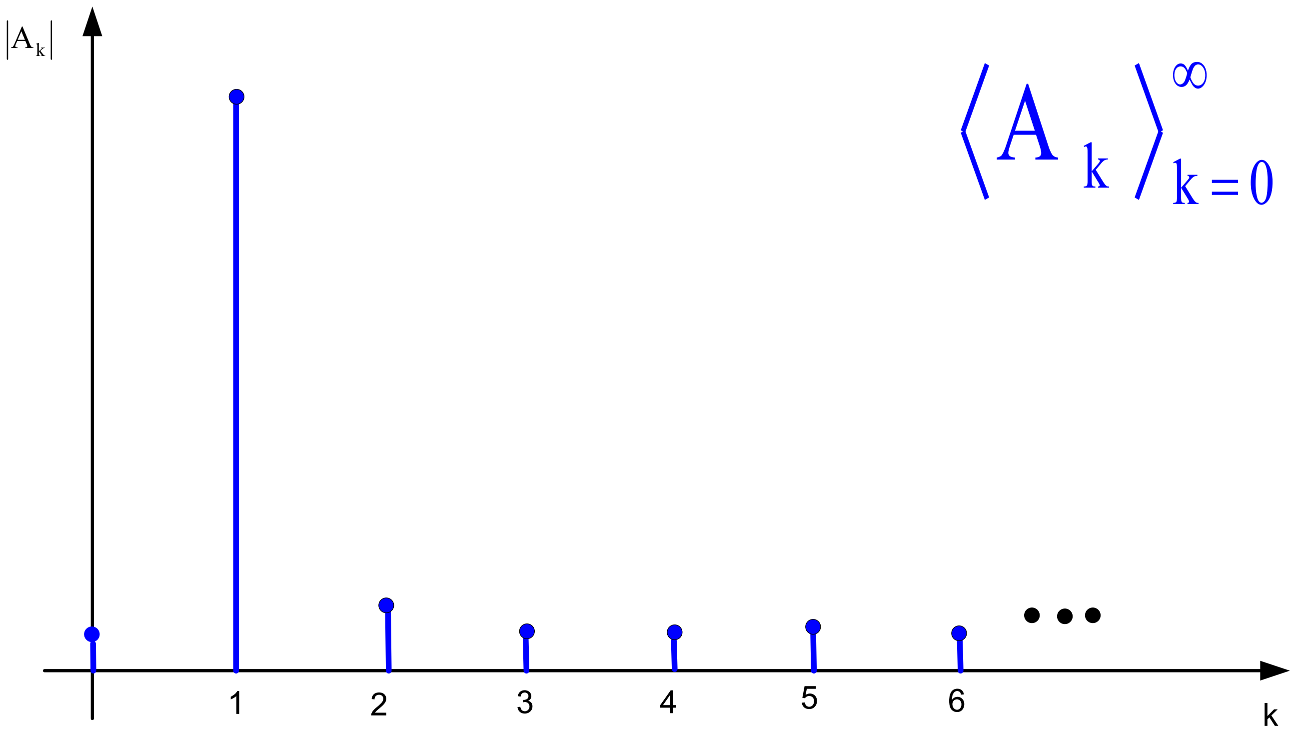
A_1 is termed the fundamental
A_k is termed the kth harmonic

Distortion Analysis



Often <u>ideal</u> response will have only fundamental present and all remaining spectral terms will vanish

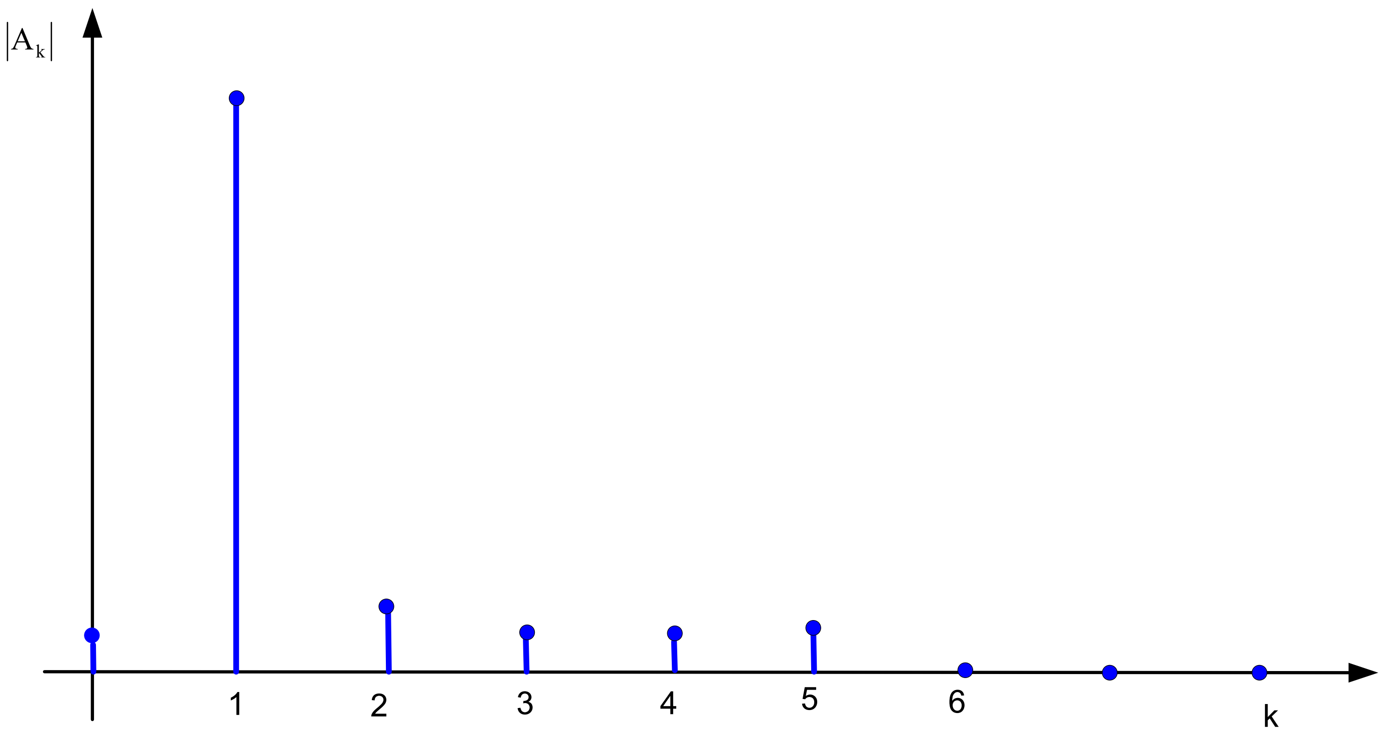
Distortion Analysis



For a low distortion signal, the 2nd and higher harmonics are generally much smaller than the fundamental

The magnitude of the harmonics generally decrease rapidly with k for low distortion signals

Distortion Analysis



Assume f(t) is periodic with period $T = \frac{1}{f}$

f(t) is band-limited to frequency $2\pi f k_X$ if $A_k=0$ for all $k>k_x$

where $\left\langle A_k \right\rangle_{k=0}^{\infty}$ are the Fourier series coefficients of f(t)

Distortion Analysis

Total Harmonic Distortion, THD

$$\mathrm{THD} = \frac{\text{RMS voltage in harmonics}}{\text{RMS voltage of fundamental}}$$

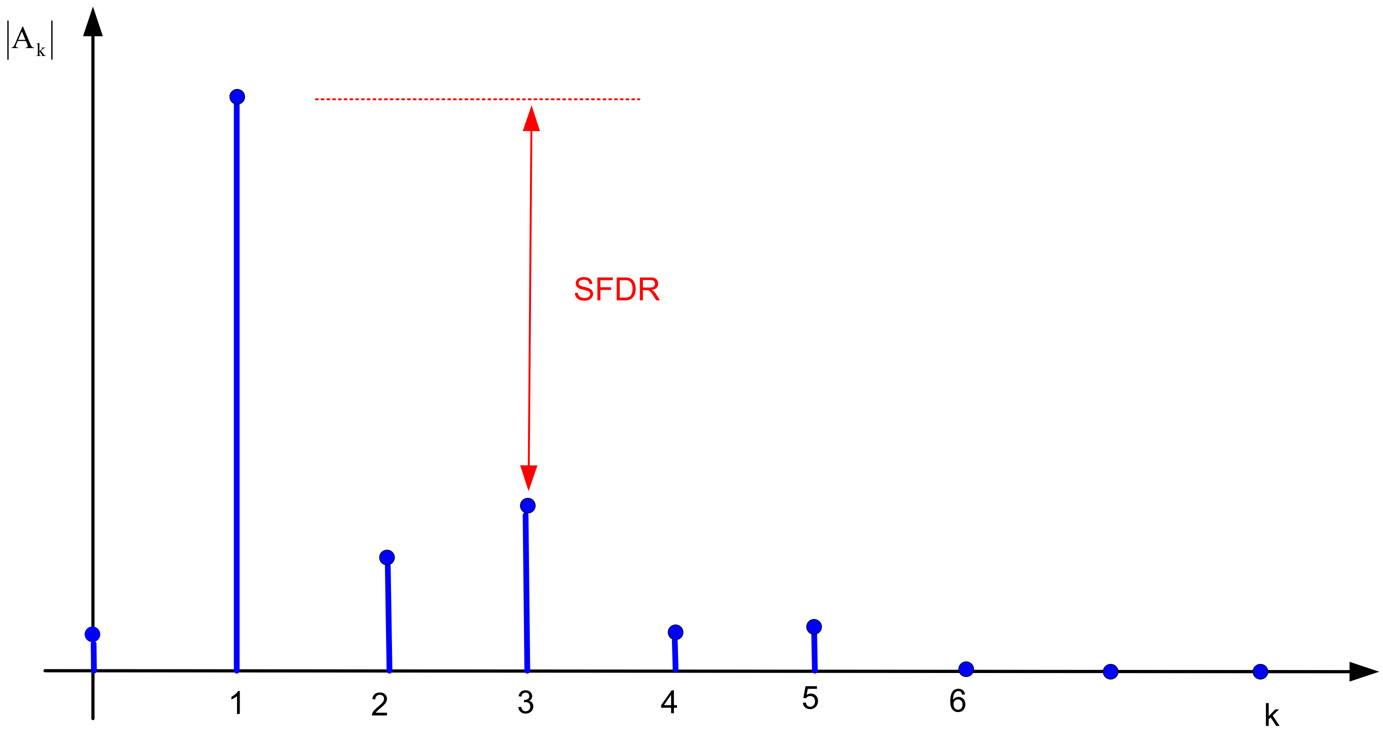
$$\mathrm{THD} = \frac{\sqrt{\left(\frac{A_2}{\sqrt{2}}\right)^2 + \left(\frac{A_3}{\sqrt{2}}\right)^2 + \left(\frac{A_4}{\sqrt{2}}\right)^2 + \ldots}}{\frac{A_1}{\sqrt{2}}}$$

$$\mathrm{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} A_k^2}}{A_1}$$

Distortion Analysis

Spurious Free Dynamic Range, SFDR

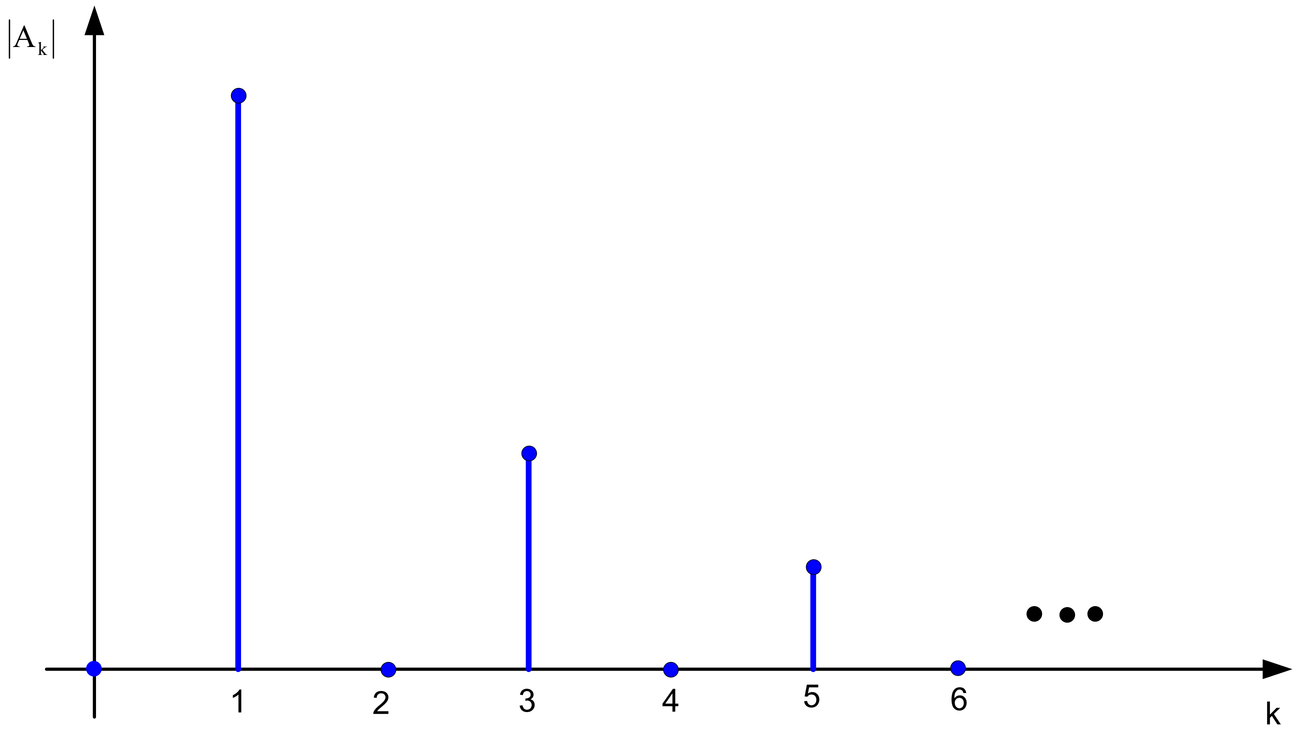
The SFDR is the difference between the fundamental and the largest harmonic



SFDR is usually determined by either the second or third harmonic

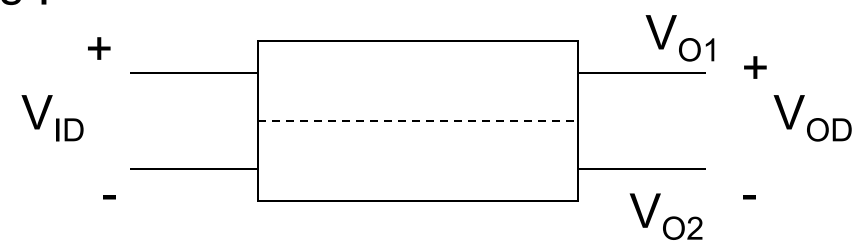
Distortion Analysis

In a fully differential symmetric circuit, all even harmonics are absent in the differential output !



Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even-order terms are absent in the Taylor's series output for symmetric differential excitations !



Proof: Expanding in a Taylor's series around V_{ID}=0, we obtain

$$V_{01} = f\left(V_{ID}\right) = \sum_{k=0}^{\infty} h_k \left(V_{ID}\right)^k$$

$$V_{02} = f\left(-V_{ID}\right) = \sum_{k=0}^{\infty} h_k \left(-V_{ID}\right)^k$$

$$V_{OD} = V_{01} - V_{02} = \sum_{k=0}^{\infty} h_k \left(V_{ID}\right)^k - \sum_{k=0}^{\infty} h_k \left(-V_{ID}\right)^k$$

$$V_{OD} = \sum_{k=0}^{\infty} h_k \left[\left(V_{ID}\right)^k - \left(-V_{ID}\right)^k\right]$$

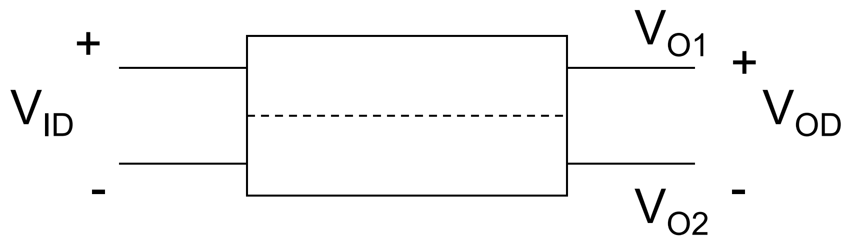
$$V_{OD} = \sum_{k=0}^{\infty} h_k \left[\left(V_{ID}\right)^k - \left(-1\right)^k \left(V_{ID}\right)^k\right]$$

When k is even, term in [ ] vanishes

Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential excitations !

Proof:



Recall:

$$\sin^n(x) = \begin{cases} \displaystyle\sum_{k=0}^{\frac{n-1}{2}} h_k \sin\big((n-2k)x\big) & \textit{for n odd} \\ \displaystyle\sum_{k=0}^{\frac{n-2}{2}} g_k \sin\big((n-2k)x+\theta_k\big) & \textit{for n even} \end{cases}$$

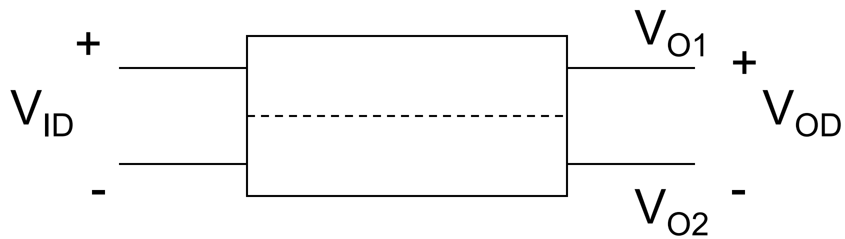
where h_k, g_k, and θ_k are constants

That is, odd powers of $\sin^n(x)$ have only odd harmonics present and even powers have only even harmonics present

Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential sinusoidal excitations !

Proof:



Expanding in a Taylor's series around V_{ID}=0, we obtain

$$V_{O1} = f\left(V_{ID}\right) = \sum_{k=0}^{\infty} h_k V_{ID}^k \qquad \text{and} \qquad V_{O2} = f\left(-V_{ID}\right) = \sum_{k=0}^{\infty} h_k (-V_{ID})^k$$

Assume V_{ID}=Ksin(ωt) W.L.O.G. assume K=1

$$V_{O1} = \sum_{k=0}^{\infty} h_k \left[\sin\left(\omega t\right)\right]^k \qquad V_{O2} = \sum_{k=0}^{\infty} h_k \left[-\sin\left(\omega t\right)\right]^k$$

$$V_{OD} = V_{O1} - V_{O2} = \sum_{k=0}^{\infty} h_k \left(\left[\sin\left(\omega t\right)\right]^k - \left[-\sin\left(\omega t\right)\right]^k\right) = \sum_{k=0}^{\infty} h_k \left(\left[\sin\left(\omega t\right)\right]^k - \left(-1\right)^k \left[\sin\left(\omega t\right)\right]^k\right)$$

Observe the even-ordered powers and hence even harmonics are absent in this last sum

Distortion Analysis

How are spectral components determined?

By integral

$$A_k = \frac{1}{\omega T}\left(\int_{t_1}^{t_1+T} f(t)e^{-jk\omega t}dt + \int_{t_1}^{t_1+T} f(t)e^{jk\omega t}dt\right)$$

or

$$a_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\sin(kt\omega)dt \qquad b_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\cos(kt\omega)dt$$

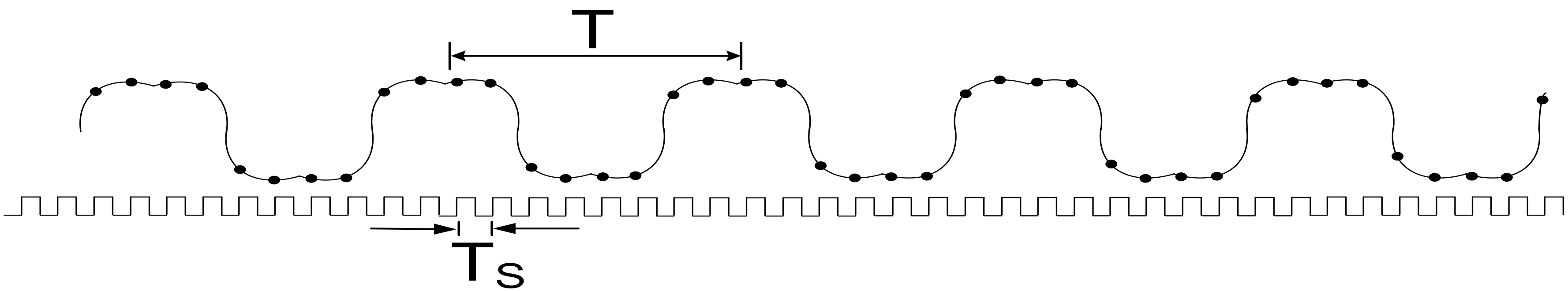
Integral is very time consuming, particularly if large number of components are required

By DFT (with some restrictions that will be discussed)

By FFT (special computational method for obtaining DFT)

Distortion Analysis

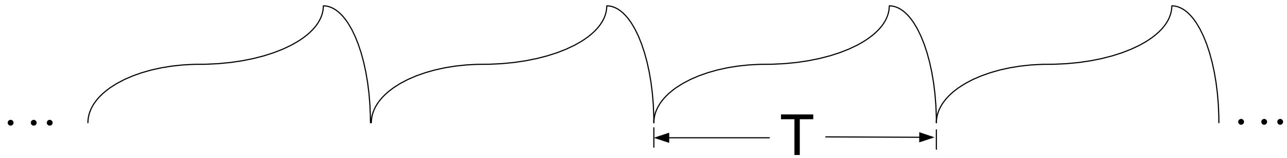
How are spectral components determined?



Consider sampling f(t) at uniformly spaced points in time T_S seconds apart

This gives a sequence of samples $\left\langle f\left(kT_s\right)\right\rangle_{k=1}^{N}$

Distortion Analysis



Consider a function f(t) that is periodic with period T

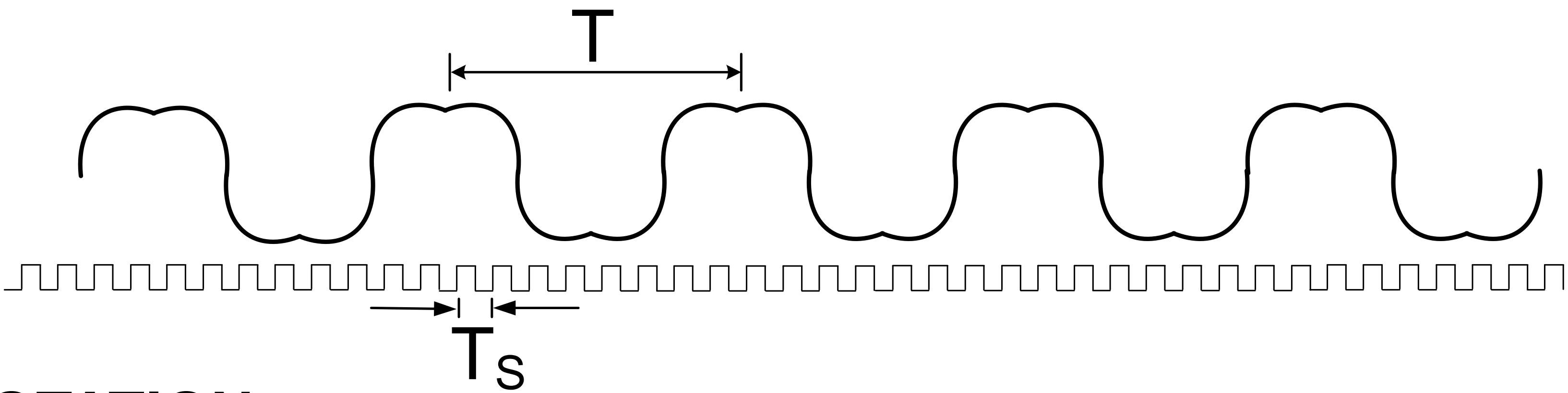
$$f(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin\left(k\omega t + \theta_k\right) \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

Band-limited Periodic Functions

Definition: A periodic function of frequency f is band

limited to a frequency f_{max} if A_k=0 for all $k > \dfrac{f_{max}}{f}$

Distortion Analysis



NOTATION:

T: Period of Excitation

T_S: Sampling Period

N_P: Number of periods over which samples are taken

N: Total number of samples

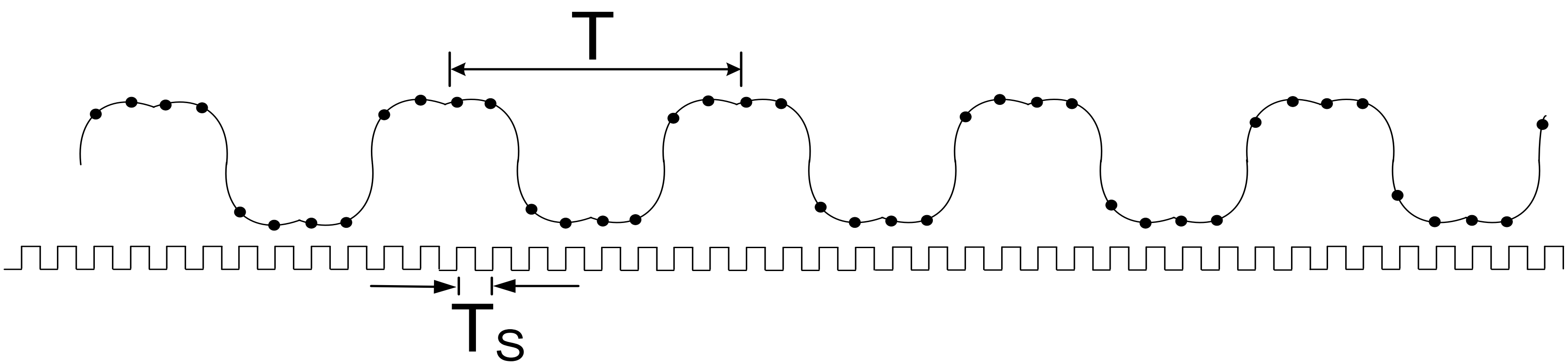
$$N_P = \frac{NT_S}{T}$$

Note: N_P is not an integer unless a specific relationship exists between N, T_S and T

$$h = \mathrm{Int}\left(\left[\frac{N}{2}-1\right]\frac{1}{N_P}\right)$$

Note: The function Int(x) is the integer part of x

Distortion Analysis



Observation : If a band-limited periodic signal is sampled at a rate that exceeds the Nyquist rate, then the Fourier Series coefficients can be directly obtained from the sampled sequence.

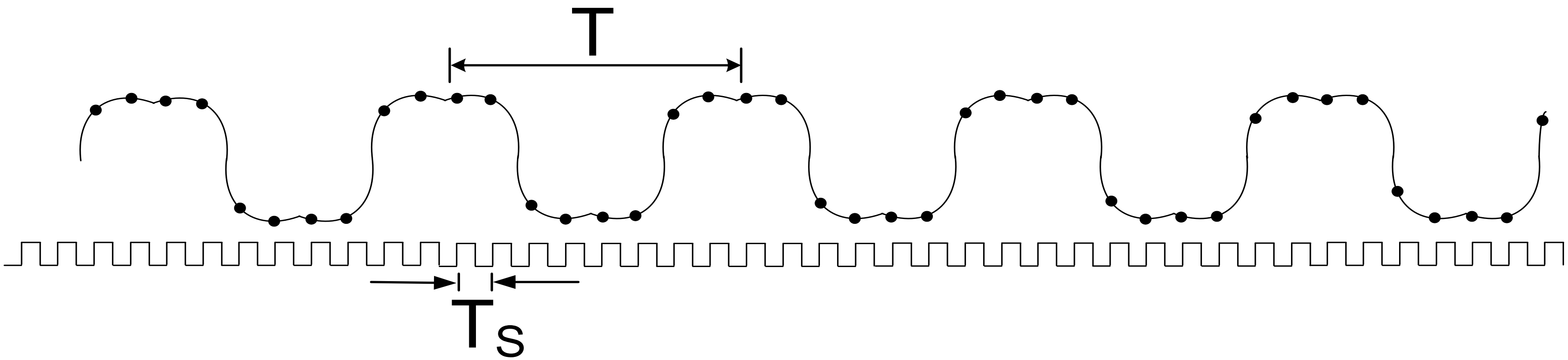
$$f(t) = A_0 + \sum_{k=1}^{N_X} A_k \sin\left(k\omega t + \theta_k\right) \qquad \omega = 2\pi \cdot f_{sig}$$

Band-limited to N_x implications $\quad A_{Nx} \neq 0 \quad A_k = 0$ for all $k > N_x$

Number of unknowns: $2N_x+1$

$f_{NYQ}=2N_x f_{sig}$ If sampled at Nyquist rate for 1 period of signal will have $2N_x$ samples

Distortion Analysis



THEOREM (conceptual) : If a band-limited periodic signal is sampled at a rate that exceeds the Nyquist rate, then the Fourier Series coefficients can be directly obtained from the DFT of a sampled sequence.

$$\left\langle x\left(kT_S\right)\right\rangle_{k=0}^{N-1} \Longleftrightarrow \left\langle X\left(k\right)\right\rangle_{k=0}^{N-1}$$

Because there is sufficient information in the sample sequence to obtain the Fourier Series coefficients

Stay Safe and Stay Healthy !

End of Lecture 27